How to Enhance Students' Mathematical Communication Ability

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Abstract We will propose a new teaching method which took in the "Mountain -Climbing Learning Method" to enhance mathematical communication ability. We practiced this method to a junior high school students grade 1-3. By the result of practice, it became clear that the power of transmitting information of students of a class which took in that learning method is higher 12%-15% than that of the students of a class which did not take in that learning method.

1. Introduction

The mathematical communication is important in order for students to learn the mathematical thinking, acquire the mathematical concept and deepen knowledge about learning contents through an interaction. For enhancing and demonstrating mathematical communication ability, the functional network of the knowledge or information about mathematics needs to be formed in his/her mind.

In this paper, we propose a learning method to enhance mathematical communication ability of students and clarify its effect.

2. Structurization of the knowledge

(1) Actual state of the systematic thinking

Learning topics and keywords which constitute the contents of the units and sections which students learn will be referred to as “learning elements” hereinafter. For the primary school students, junior high school students, high school students, undergraduate students, novice teachers and experienced teachers. We examined how they grasp and understand the structural relations of the contents of a textbook in the primary school, junior high school, high school, university which they had learn [1)].

The primary school students and junior high school students only understand the partial relationship among learning elements, rather than the holistic view of learning elements. The high school students and undergraduate students are more capable than junior high school students in understanding of the sequence of learning elements, but not to the holistic relationship. Similarly, the novice teachers understand the sequence of learning elements to a considerable degree, but still not to the holistic relationship. The experienced teachers can systematically understand the holistic relationship of learning elements.

For the degree of understanding of the structural relations, it was shown using five scale score evaluation. The primary school students, junior high school students, high school students, undergraduate students and novice teachers obtained a score of 2-3. These results show that students do not understand structure of leaning contents in the present school education.

Figure 1 (a), (b), (c) and (d) show the understanding of the structural relations among the learning elements of the junior high school students, undergraduate students, novice teachers and experienced teachers for junior high school 2nd grade "linear-function." An arrow line means that not less than 50% of the examinee who judged that it was related.

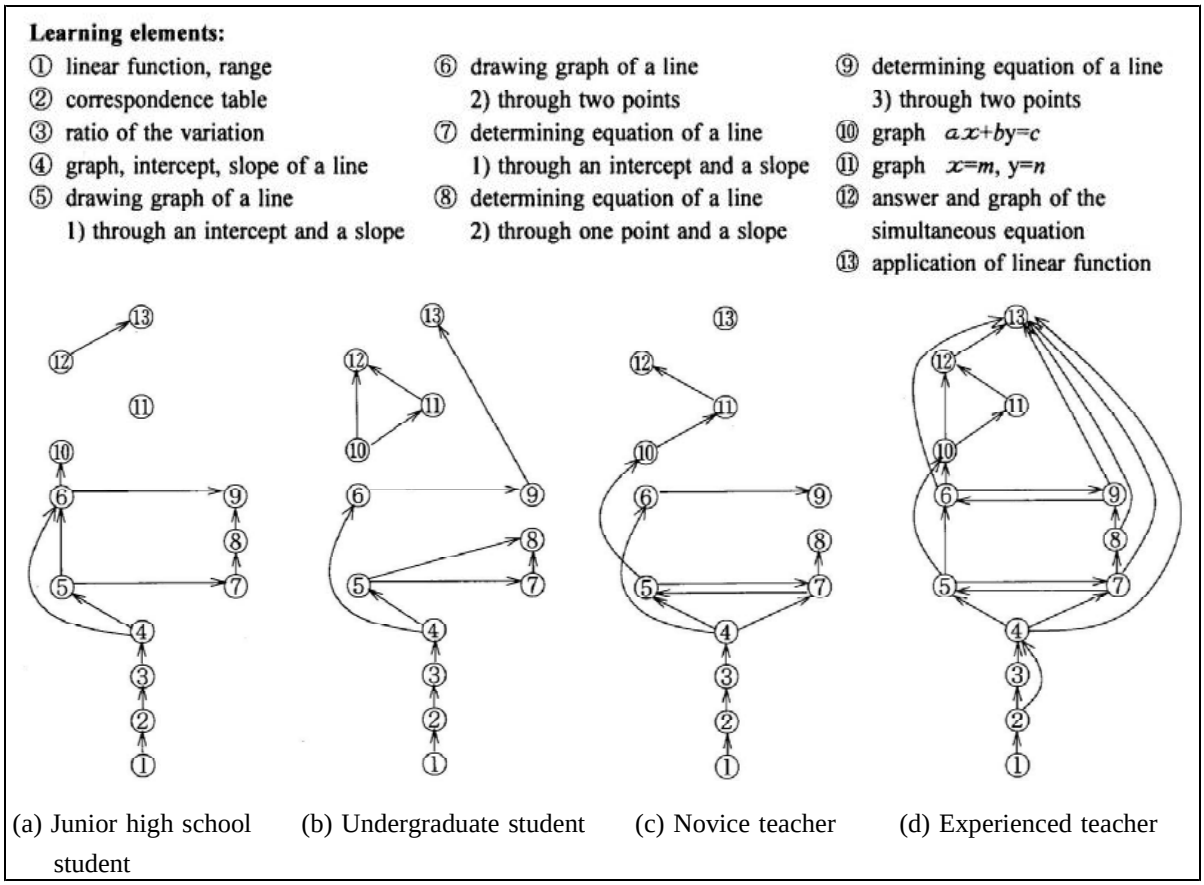

Figure 1 Understanding of the structural relations among the learning elements (Junior high school 2nd grade "Linear Function")

(2) Cognitive structure of an experienced teacher and a student

We think about a cognitive structure model of an experienced teacher and a student as shown in Figure 2 (a).

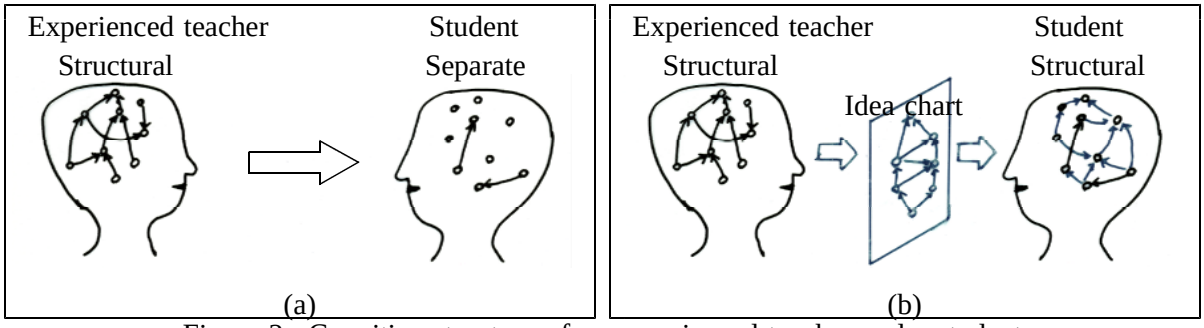

Figure 2 Cognitive structure of an experienced teacher and a student

As for a experienced teacher, he grasps and understands teaching elements and contents systematically and structurally through the teaching guide, reference book, teaching experience and so on. In other words, we can think that the experienced teacher forms the functional network of the knowledge and the information in his mind.

On the other hand, the students are the learners, who are confined to the understanding of each

learning item. They do not grasp and understand the systematical and functional relationship among the whole learning contents. In other words, the functional network of the knowledge and the information are not formed in the student's mind.

In case of the student's knowledge is separated and not formed systematically, the student may be able to solve a problem concerning each learning items, but may not be able to solve a integrated problem and a applied problem which need the synthetic and overall knowledge. Moreover, the student may not have interest of the learning contents. In case of the functional network of the knowledge and the information are formed in the mind, they use knowledge one after another from their mind.

Therefore it is very important to form functional network in the mind through structuring knowledge and information for enhancing mathematical communication ability.

3. Making the base for enhancing mathematical communication ability

In order for students to form the functional network of the knowledge and the information in their mind, a teacher gives students a chart which is drawn based on the teacher's idea about the structure of learning contents of a unit, and makes the students use it as a learning material to help them in their understanding (See Figure 1 (b)).

This chart is called "Structural Learning Chart" or "Concept-map." The teaching method using the chart is as follows;

(1) Preparation before the class by teacher

The teachers analyze the contents of the materials in each unit, and they construct a "Structural Learning Chart" before the class. Next, based on the "Structural Learning Chart," they construct an "Explanatory Form of Arrows" and a "Self-Diagnosis Sheet."

The following is the concrete procedure of making the three teaching materials;

(A) Constructing of a "Structural Learning Chart"

Step 1: Based on the textbook or the teachers' manual and their experiences, select some 10 to 20 “learning elements” which are necessary components of a unit and write down on a paper.

Step 2: Examine the relation between each pairs of selected "learning elements."

The teachers examine any two learning elements to see whether they have any relation such as presupposive, hierarchical, causal, logical, inclusive, subsumptive and orderly.

Step 3: Arrange all "learning elements" in hierarchical network depending on some relations between them.

Step 4: Draw arrow lines to indicate the relation between two learning elements.

The arrow line may be drawn in bottom up/parallel depending on the relation.

Step 5: Correct any mistake in the "Structural Learning Chart" by reexamining the arrow lines appeared on the chart.

Figure 3 shows the "Structural Learning Chart" of the "Linear Function" for junior high school grade 2.

(B) Making an "Explanatory Form of Arrows"

The teacher makes an "Explanatory Form of Arrows" for students to fill in the relations in dictated by the arrows in the "Structural Learning Chart."

Table 1 shows the "Explanatory Form of Arrows."

(C) Making a "Self-Diagnosis Sheet"

The teacher makes a "Self-Diagnosis Sheet" where students write what they feel as unknown and wish to study deeper in class.

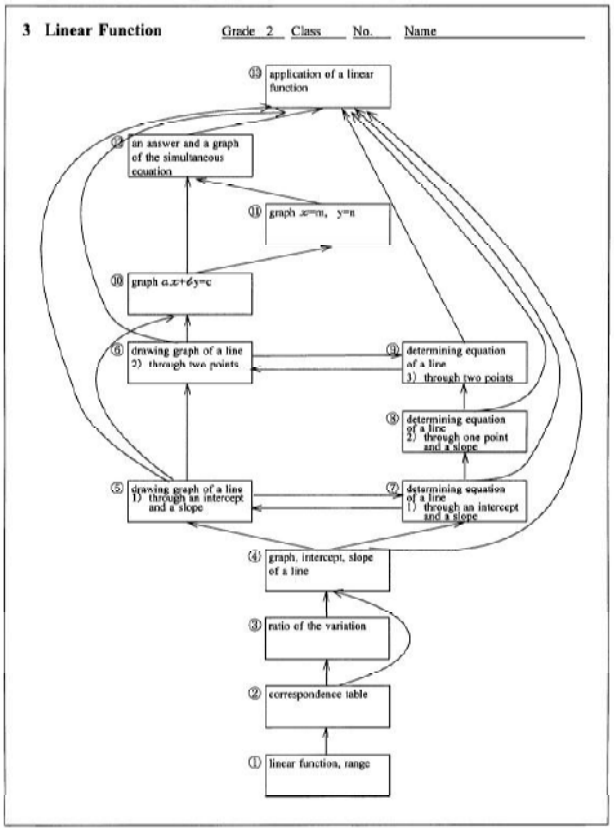

Figure 3 Structural Learning Chart (Junior high school 2nd grade "Linear Function")

Table 1 Explanatory Form of Arrows

Explanatory Form of Arrows

3 Linear Function Grade 2 Class No. Name

Arrow line	Reason for relationship between two learning elements
①→②	
②→③	
②→④	
③→④	
④→⑤	
④→⑦	
④→⑬	
⑤→⑥	
⑤→⑦	
⑤→⑩	
⑤→⑬	
⑥→⑨	
⑥→⑩	
⑥→⑬	
⑦→⑤	
⑦→⑧	
⑦→⑬	
⑧→⑨	
⑧→⑬	
⑨→⑥	
⑨→⑬	
⑩→⑪	
⑩→⑫	
⑪→⑫	
⑫→⑬	

Table 2 Self-Diagnosis Sheet

Self-Diagnosis Sheet

3 Linear Function Grade 2 Class No. Name

◎ : I understood well. ○ : I understood a little. △ : I was not able to understand.

Learning element	Self-evaluation	- I felt it wonderful. - I would like to investigate more deeply.
① linear function, range		
② correspondence table		
③ ratio of the variation		
④ graph, intercept, slope of a line		
⑤ drawing graph of a line 1) through an intercept and a slope		
⑥ drawing graph of a line 2) through two points		
⑦ determining equation of a line 1) through an intercept and a slope		
⑧ determining equation of a line 2) through one point and a slope		
⑨ determining equation of a line 3) through two points		
⑩ graph $ax+by=c$		
⑪ graph $x=m$, $y=n$		
⑫ answer and graph of the simultaneous equation		
⑬ application of linear function		

Table 2 shows the "Self-Diagnosis Sheet."

(2) Teaching sequence

In this section, the teaching sequence will be described using the following three forms of preparations teachers prepared prior to the class: "Structural Learning Chart," "Explanatory Form of Arrows" and "Self-Diagnosis Sheet."

Step 1: At the beginning of the class, the teacher hands out the "Structural Learning Chart," "Explanatory Form of Arrows" and "Self-Diagnosis Sheet" to all students.

Step 2: The teacher instructs students to enter into the margin of the "Structural Learning Chart" summarizing of what they have learned about individual learning elements. The students write down the following sorts of items in the margin of the "Structural Learning Chart":

- Explanation of the learning elements they have learned in the class.
- Mathematical formulas.
- Typical exercises or questions and answers they produced on their own (i.e. problem posing).

In this process, the teacher aims to make the students understand the basic and fundamental learning contents tightly by organizing that activity in the learning of contents.

Photographs 1 shows a scene where a primary school students write down the main points of a learning element in the "Structural Learning Chart.

Photograph 1 A scene where students write down the main points into space of the "Structural Learning Chart" (Primary school grade 6)

Step 3: Using the Structural Learning Chart as reference, the students fill in the reasons for the arrows in the "Explanatory Form of Arrows."

The "Explanatory Form of Arrows" is drawn to help students clearly understand the contents and meaning of each learning element and to grasp the relationships among the learning elements.

Step 4: The Students write down their questions and the problems they wish to explore further on the "Self-Diagnosis Sheet."

The "Self-Diagnosis Sheet" aims to raise the problem consciousness of students about the subject matter and to improve their ability and motivation to find new and worthwhile questions on their own by making them carefully read the contents in order to gain a more deeply understanding.

Step 5: At the end of a teaching unit, when the subject matter is summarized, the teacher again asks students to reexamine the contents of the "Structural Learning Chart" to present reports and hold discussions on a range of issues, such as the meaning of each learning element, the structural relationships existing in the learning contents as a whole and the outline or plot (story) of those contents.

The presentations and the discussions on learning contents allow students to verbally externalize the conceptual structure of the teaching unit that they have in their mind. Those help students to acquire a more detailed, contextualized and systematic understanding of the contents to form a functional network of knowledge and information and improve their mathematical communication ability.

Photographs 2 shows the scene of the discussing the structural relations of the learning contents of a unit of the primary school students and the junior high school students.

(a) Primary school grade 5

(b) Junior high school grade 1

Photograph 2 Discussion of the structural relations among the learning elements of the learning contents

We call the learning method using the "Structural Learning Chart" that follows **Steps 1** to **5** the "Mountain-Climbing Learning Method [2)]."

The aims of the "Mountain-Climbing Learning Method" are as follows;

- To understand tightly basic and fundamental contents.
- To activate students' structural and systematic thinking.
- To make the functional network of the knowledge and the information in students' mind.
- To activate students' creative thinking.
- To raise students' interesting for doing mathematics.

The Mountain-Climbing Learning Method suits all students of the low, middle and high scholastic ability and is highly effective.

4. Class practice and its results

(1) Grades and topics

Grade / Number of students	Topics	Class hours
Junior high school grade 1 / 153 (Group A 76, Group B 77)	"6 Space Figure"	14 hours
Junior high school grade 2 / 136 (Group A 68, Group B 68)	"7 Similarity"	14 hours
Junior high school grade 3 / 148 (Group A 73, Group B 75)	"5 Circles and Circumference"	16 hours

(2) Teaching Method

Group A: The class which take in the "Mountain-Climbing Learning Method."

Group B: The class which performed usual lesson by using a textbook.

The teacher gave the "Structural Learning Chart," "Explanatory Form of Arrows" and "Self-Diagnosis Sheet" to students in Group A, and "Self-Diagnosis Sheet" to students in Group B.

Group A:	Class by using a textbook + Mountain-Climbing Learning Method (11 or 13 hours)	Solving subject(2h)	Presentation of results (1h)
Group B:	Class by using a textbook (11 or 13 hours)	Solving subject(2h)	Presentation of results (1h)

(3) Setup of research subjects

The teacher selects 6-12 doubted or interesting topics written on the student's "Self-Diagnosis

Sheet."

Table 3 shows the major subjects of the students that were written on their "Self-Diagnosis Sheet."

Table 3 The major subjects of the students that were written on their "Self-Diagnosis Sheet"

Grade 1	- I would like to investigate in detail about a cubic section. - I would like to find various body of revolution. - Why is a plane decided by three points? - I would like to know more about "skew." - How many does a cubic developments exist? - How is an angle at which two lines/ planes measured? - I would like to know Euler's theorem in detail. - Why are there only five regular polyhedrons?
Grade 2	- Are there any quadrilateral similarity condition? - I would like to actually find the size of a thing using similarity. - I would like to know the origin of a similar sign. - I would like to know about the middle point connection theorem of various polygons. - I would like to find the centroid of various figures. - I would like to find the way of bisecting a triangle in a straight line. - Is there any property of the similarity except having learned? - I would like to investigate about seven-wise-men.
Grade 3	- I would like to know the relation among a circle, a straight line and a triangle. - Does an inscribed circle exist in any triangles? - I would like to count the number of the intersections of two circles in more detail. - I would like to investigate whether a circle is inscribed in a polygon. - How is the radius of a triangular inscribed circle calculated? - If it considers that the earth is a circle, what kind of thing can say? - I would like to investigate about the person who found the theorem. - I would like to investigate the relation of two corners which the quadrangle inscribed in a circle adjoins.

The students selected a subject which they would like to research. The teacher made some group. The groups are formed by students who have same subject. The students of each group plugged away at their subject through examining the books in the library, experimenting in ground and discussing with teacher and solved their subjects.

6 groups were organized per class in grade 1, 7-8 groups were organized per class in grade 2, 8-12 groups were organized per class in grade 3.

(4) Presentation about research results

The students of each group made presentation about the research results for their subject. They explained by using OHP sheets or large papers which were written their points. The students who were hearing the presentation did some questions to the presenter repeatedly until they could understand the content of their explanation.

Photographs 3(a) shows a scene of the researching subject in the library and 3(b) shows a scene of the presentation about research results of junior high school students.

(a) Researching subject in the library

(b) Presentation about research results

Photographs 3 A scene of the researching subject (Junior high school student grade 3)

The student's impression for solving process of research tasks and the presentation of the results were as follows;

- While I was investigating, I have understood the meaning of some formula which did not understand before that.
- When I investigated at a library, I found various ways of solving.
- I can say now the reason for why it become such a result.
- I understood well what kind of way of solving and thinking of others used. And I came to consider various ways of solving.
- I was able to know the content except having learned, therefore I got a broader point of view.

Some contents investigated by the 1st grade students in a junior high school had reached the learning contents which they were learned the 3rd grade in a junior high school, and some contents investigated by the 3rd grade students in a junior high school had reached the learning contents which they were learned the 1st-2nd grade in a high school.

(5) Evaluation method of the power of transmitting information

In communication in the presentation about research results, the students need the power of transmitting information for explaining the content intelligibly for others through organizing knowledge and information. Based on these things, we set up the following evaluation viewpoints about the power of transmitting information.

The first, we classify the power of transmitting information into the "Organization of Information (descriptive expression)" and "Transmission of Information (verbal expression)." The second, we classified those evaluation viewpoints into following detailed viewpoints that were considered important.

Organization of Information (Descriptive expression)
- 1) Ingenuity of expression
- 2) Organization of the contents

Transmissin of information (Verbal expression)
- 3) Logical reason
- 4) Clarity of the argument points
- 5) Consensus with participants

We gave 20 as the full marks of each viewpoints 1)-5). The total full mark is 100.

(6) Results of evaluation of the power of transmitting information

Table 4 shows the average of evaluation of the power of transmitting information for the students who belong to Groups A and B in the junior high school grade 1-3.

Table 4 The average of evaluation of the power of transmitting information

viewpoints / Group		Organization of information (Descriptive expression)		Transmission of information (Verbal expression)			Total
		Ingenuity of expression	Organization of the contents	Logical reason	Clarity of the argument points	Consensus with participants	
Grade 1	A	14.3	14.1	13.5	13.1	13.4	68.4
	B	10.7	11.7	10.8	10.0	10.3	53.5
Grade 2	A	15.9	15.7	14.7	14.3	14.5	75.1
	B	11.5	12.9	12.1	12.5	12.3	61.3
Grade 3	A	16.1	16.0	15.3	15.0	15.4	77.8
	A	11.6	13.8	13.5	13.3	13.0	65.2

It became clear that the average of evaluation of the power of transmitting information of group A which took in the "Mountain-Climbing Learning Method" is higher 12%-15% than that of group B in all grades.

However, at the beginning of the unit, the average of the achievement test of group A was 57.6 and group B was 54.5 in the junior high school grade 1, group A was 51.4 and group B was 55.6 in the junior high school grade 2 and group A was 40.5 and group B was 44.7 in the junior high school grade 3.

5. Conclusion

In order for students to enhance mathematical communication ability, it is important that the students understand tightly the basic and fundamental contents, activate the structural and systematic thinking and make the functional network of the knowledge and the information in their mind.

In this paper, we took in the "Mountain-Climbing Learning Method" for enhancing them sufficiently.

By the results of the practice for the students of the junior high school grade 1-3, it became clear that the mark of the transmitting information of group A which took in the "Mountain-Climbing Learning Method" is higher 12%-15% than the mark of group B which performed the usual lesson by using a textbook.

References

1) Noboru Saito (1995), A research of development of the structural thinking and understanding in mathematics learning, The Bulletin of Japanese Curriculum Research and Development, Vol.18, No.2, pp.33-39.

2) Noboru Saito (2004), Mountain-Climbing Learning Method, Meijitosho.